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RADIATION FROM A MOVING MAGNETON

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1. The rate of radiation of energy from a ring of electrons revolving in a circular orbit and from various other distributions of moving electric charges and magnetic poles has been calculated by G. A. Schott,[1] who finds that the rate of radiation of energy is almost invariably positive. This is certainly true in the case of a single electric pole describing a circular orbit as is indicated by the well known formulae of Larmor and Liénard for the rate of radiation. Thus electromagnetic theory in its present form lends no support to Bohr's idea of non-radiating orbits.

A steady distribution such as a Parson magneton which consists of a complete ring of electric charges following one another round the ring at a constant speed will evidently give no radiation when the ring is stationary as a whole, but as Schott remarks the ring may be expected to radiate energy when its centre has an acceleration.

Schott's results are so important that it is desirable that they should be confirmed by an independent method and an attempt has been made to devise a method by which the rate of radiation from a moving electric pole and magnetic doublet may be readily calculated. In two important cases we have confirmed Schott's surmise that the rate of radiation is positive.

2. Starting with the case of an *electromagnetic doublet*, i.e. an electric doublet and magnetic doublet which move together, we determine the electric force E and the magnetic force H from the equations

$$M \equiv H + iE = i \operatorname{rot} L$$

$$L = \frac{\partial G}{\partial t} + ic \operatorname{rot} G, \qquad G = \frac{1}{p} g(\tau)$$

$$4\pi g = q + ip - \frac{1}{c}(v \times p) + \frac{i}{c}(v \times q)$$

$$\nu = \xi'(\tau)[x - \xi(\tau)] + \eta'(\tau)[y - \eta(\tau)] + \zeta'(\tau)[z - \zeta(\tau)] - c^2(t - \tau)$$
$$\equiv r[(v \cdot s) - c]$$
$$r^2 = [x - \xi(\tau)]^2 + [y - \eta(\tau)]^2 + [z - \zeta(\tau)]^2 = c^2(t - \tau)^2. \qquad t \geq \tau.$$

In these equations v denotes the velocity at time τ of the moving point P whose coordinates at this instant are $\xi(\tau)$, $\eta(\tau)$, $\zeta(\tau)$; x, y, z are the coordinates of an arbitrary point Q; s is a unit vector in the direction of the line PQ; p and q are vectors representing the electric and magnetic moments at time τ; t is the time, and c the velocity of light.

From these equations we find that

$$E \times H = c^2 \frac{r^4}{\nu^4} s[g^* \cdot g_0^* - (s \cdot g^*)(s \cdot g_0^*) - i\{s \cdot (g^* \times g_0^*)\}]$$

where g^* is a certain complex vector and g^*_0 the conjugate complex vector. A similar expression may be obtained in the case of an electromagnetic pole and also in the case of the combination of an electromagnetic pole and an electromagnetic doublet. In the latter case the appropriate expression for g^* is

$$g^* = \frac{1}{\nu} b - 3\frac{r}{\nu^2}(s \cdot v')g' + \frac{3r^2}{\nu^3}(s \cdot v')^2 g - \frac{r}{\nu^2}(s \cdot v'')g$$

where

$$b = g'' + mv' + i\frac{m}{c}(v \times v'),$$

and

$$m = h + ie.$$

Here $4\pi e$ and $4\pi h$ are the electric and magnetic charges associated with the pole and primes denote differentiations with respect to τ.

Calculating the rate of radiation I across a very large sphere whose centre is at the moving point P we find that I may be represented by the real part of the following expression

$$\pi c^3 \Big[\frac{8}{3}\frac{c^2 + 2v^2}{(c^2 - v^2)^4}(b \cdot b_0) - \frac{8}{(c^2 - v^2)^4}(v \cdot b)(v \cdot b_0) + \frac{16}{5}\frac{c^2 + v^2}{(c^2 - v^2)^5}\Big\{12(v \cdot v')(b \cdot g_0')$$
$$+ 3v'^2(g' \cdot g_0') + 4(v \cdot v'')(b \cdot g_0) + 2v'^2(b \cdot g_0) + 2(v' \cdot v'')(g' \cdot g_0) + \frac{1}{3}v''^2(g \cdot g_0)\Big\}$$
$$+ \frac{8}{5}\frac{3c^2 + 2v^2}{(c^2 - v^2)^6}\Big\{24(v \cdot v')^2(g' \cdot g_0') + 16(v \cdot v')^2(b \cdot g_0) + \frac{144}{7}v'^2(v \cdot v')(g' \cdot g_0)$$

$$+16(v\cdot v')(v\cdot v'')(g'\cdot g_0)+\frac{9}{7}v'^4(g\cdot g_0)+\frac{16}{7}(v\cdot v'')v'^2(g\cdot g_0)+\frac{32}{7}(v\cdot v')(v'\cdot v'')(g\cdot g_0)$$

$$+\frac{8}{3}(v\cdot v'')^2(g\cdot g_0)\Big\}+\frac{64}{7}\frac{2c^2+v^2}{(c^2-v^2)^7}\Big\{24(v\cdot v')^3(g'\cdot g_0)+9v'^2(v\cdot v')^2(g\cdot g_0)$$

$$+8(v\cdot v'')(v\cdot v')^2(g\cdot g_0)\Big\}-\frac{8}{5}\frac{1}{(c^2-v^2)^4}\Big\{6(v\cdot b)(v'\cdot g_0')+6(v\cdot g_0')(v'\cdot b)$$

$$+3(v'\cdot g')(v'\cdot g_0')+2(v'\cdot b)(v'\cdot g_0)+(v'\cdot v'')(g'\cdot g_0)+(v''\cdot g')(v'\cdot g_0)$$

$$+(v'\cdot g')(v''\cdot g_0)+\frac{1}{3}(v''\cdot g)(v''\cdot g_0)+2(v\cdot b)(v''\cdot g_0)+2(v\cdot g_0)(v''\cdot b)\Big\}$$

$$-\frac{32}{5}\frac{1}{(c^2-v^2)^5}\Big\{12(v\cdot v')(v\cdot g_0')(v\cdot b)+6(v\cdot v')(v\cdot g')(v'\cdot g_0')+6(v\cdot v')(v'\cdot g')(v\cdot g_0')$$

$$+3v'^2(v\cdot g')(v\cdot g_0')+2v'^2(v\cdot b)(v\cdot g_0)+4(v\cdot v')(v\cdot b)(v'\cdot g_0)+4(v\cdot v')(v'\cdot b)(v\cdot g_0)$$

$$+2(v'\cdot v'')(v\cdot g')(v\cdot g_0)+2(v\cdot v')(v\cdot g')(v''\cdot g_0)+2(v\cdot v'')(v\cdot g_0)(v'\cdot g')$$

$$+2(v\cdot v'')(v\cdot g')(v'\cdot g_0)+2(v\cdot v')(v\cdot g_0)(v''\cdot g')+\frac{4}{3}(v\cdot v'')(v\cdot g)(v''\cdot g_0)$$

$$+\frac{1}{3}v''^2(v\cdot g)(v\cdot g_0)+\frac{18}{7}v'^2(v\cdot g')(v'\cdot g_0)+\frac{18}{7}v'^2(v\cdot g_0)(v'\cdot g')+\frac{4}{7}(v\cdot v'')(v'\cdot g)(v'\cdot g_0)$$

$$+\frac{4}{7}(v'\cdot v'')(v'\cdot g_0)(v\cdot g)+\frac{4}{7}(v'\cdot v'')(v'\cdot g)(v\cdot g_0)+\frac{4}{7}(v\cdot v')(v'\cdot g)(v''\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')(v'\cdot g_0)(v''\cdot g)+4(v\cdot v'')(v\cdot b)(v\cdot g_0)+\frac{36}{7}(v\cdot v')(v'\cdot g')(v'\cdot g_0)$$

$$+\frac{4}{7}(v\cdot g)(v''\cdot g_0)v'^2+\frac{9}{14}v'^2(v'\cdot g)(v'\cdot g_0)\Big\}-64\frac{1}{(c^2-v^2)^6}\Big\{3(v\cdot v')^2(v\cdot g')(v\cdot g_0')$$

$$+2(v\cdot v')^2(v\cdot b)(v\cdot g_0)+2(v\cdot v')(v\cdot v'')(v\cdot g')(v\cdot g_0)+\frac{1}{3}(v\cdot v'')^2(v\cdot g)(v\cdot g_0)$$

$$+\frac{18}{7}(v\cdot v')^2(v\cdot g')(v'\cdot g_0)+\frac{18}{7}(v\cdot v')^2(v\cdot g_0)(v'\cdot g')+\frac{4}{7}(v\cdot v')(v\cdot v'')(v\cdot g)(v'\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')(v\cdot g)(v\cdot g_0)(v'\cdot v'')+\frac{4}{7}(v\cdot v')(v\cdot v'')(v\cdot g_0)(v'\cdot g)+\frac{18}{7}v'^2(v\cdot v')(v\cdot g')(v\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')^2(v\cdot g)(v''\cdot g_0)+\frac{2}{7}v'^2(v\cdot v'')(v\cdot g)(v\cdot g_0)+\frac{9}{56}v'^4(v\cdot g)(v\cdot g_0)$$

$$+\frac{9}{14}(v\cdot v')^2(v'\cdot g)(v'\cdot g_0)+\frac{9}{7}v'^2(v\cdot v')(v\cdot g)(v'\cdot g_0)+\frac{192}{7}\{24(v\cdot v')^3(v\cdot g')(v\cdot g_0)$$

$$+8(v\cdot v')^2(v\cdot v'')(v\cdot g)(v\cdot g_0)+9v'^2(v\cdot v')^2(v\cdot g)(v\cdot g_0)+12(v\cdot v')^3(v\cdot g)(v'\cdot g_0)\}$$

$$-576\frac{1}{(c^2-v^2)^8}(v\cdot v')^4(v\cdot g)(v\cdot g_0)-i\pi c^2\Big[\frac{4}{15}\frac{5c^2+v^2}{(c^2-v^2)^4}\{5[v\cdot(b\times b_0)]$$

$$+6[v'\cdot(b\times g_0')]+2[v''\cdot(b\times g_0)]\}+\frac{4}{5}\frac{7c^2+v^2}{(c^2-v^2)^5}\{4(v\cdot v'')[v\cdot(b\times g_0)]$$

$$+ 12\,(v\cdot v')\,[v\cdot(b\times g_0')] + 6\,(v\cdot v')\,[v'\cdot(g'\times g_0')] + 3v'^2\,[v\cdot(g'\times g_0')]$$
$$+ 4\,(v\cdot v')\,[v'\cdot(b\times g_0)] + 2v'^2\,[v\cdot(b\times g_0)] + 2\,(v\cdot v')\,[v''\cdot(g'\times g_0)]$$
$$+ 2\,(v\cdot v'')\,[v'\cdot(g'\times g_0)] + 2\,(v'\cdot v'')\,[v\cdot(g'\times g_0)] + \frac{2}{3}\,(v\cdot v'')\,[v''\cdot(g\times g_0)]$$
$$+ \frac{1}{3}\,v''^2\,[v\cdot(g\times g_0)] + \frac{18}{7}\,v'^2\,[v'\cdot(g'\times g_0)] + \frac{4}{7}\,(v'\cdot v'')\,[v'\cdot(g\times g_0)]$$
$$+ \frac{2}{y}\,v'^2[v''\cdot(g\times g_0)] + \frac{16}{5}\,\frac{9c^2+v^2}{(c^2-v^2)^6}\,\{6\,(v\cdot v')^2[v\cdot(g'\times g_0')] + 4\,(v\cdot v')^2[v\cdot(b\times g_0)]$$
$$+ 4\,(v\cdot v')\,(v\cdot v'')\,[v\cdot(g'\times g_0)] + \frac{2}{3}\,(v\cdot v'')^2[v\cdot(g\times g_0)] + \frac{9}{7}\,v'^2(v\cdot v')\,[v'\cdot(g\times g_0)]$$
$$+ \frac{9}{28}\,v'^4\,[v\cdot(g\times g_0)] + \frac{4}{7}\,(v\cdot v')^2\,[v''\cdot(g\times g_0)] + \frac{8}{7}\,(v\cdot v')\,(v\cdot v'')\,[v'\cdot(g\times g^0)]$$
$$+ \frac{8}{7}\,(v\cdot v')\,(v'\cdot v'')\,[v\cdot(g\times g_0)] + \frac{4}{7}\,v'^2\,(v\cdot v'')\,[v\cdot(g\times g_0)]$$
$$+ \frac{16}{7}\,\frac{11c^2+v^2}{(c^2-v^2)^7}\,\{24\,(v\cdot v')^3\,[v\cdot(g'\times g_0)] + 12\,(v\cdot v')^2\,(v\cdot v'')\,[v\cdot(g\times g_0)]$$
$$+ 6\,(v\cdot v')^3\,[v'\cdot(g\times g_0)] + 9\,v'^2\,(v\cdot v')^2\,[v\cdot(g\times g_0)]\}$$
$$+ \frac{288}{7}\,\frac{13c^2+v^2}{(c^2-v^2)^8}\,(v\cdot v')^4[v\cdot(g\times g_0)].$$

3. In the special case of a magneton of charge $4\pi e$ and moment $4\pi k$ describing a circular orbit at a constant speed and in such a way that the axis of the magneton is always perpendicular to the plane of the orbit, we find that if k is constant the rate of radiation of energy is

$$\frac{8}{3}\,\pi ce^2\,\frac{v'^2}{(c^2-v^2)^2} + \frac{8}{3}\,\pi c^2ek\,\frac{v'^3}{v(c^2-v^2)^3} + \frac{16}{15}\,\pi ck^2\,\frac{v'^4}{v^2}\,\frac{2c^2+3v^2}{2(c^2-v^2)^4}$$

If e, v and v' are given the minimum value of this positive quantity is found to be

$$\frac{2\pi}{3}\,\frac{ce^2v'^2}{(c^2-v^2)^2}\,\frac{c^2+4v^2}{2c^2+3v^2}$$

and is thus about $\frac{1}{3}$ of the rate of radiation from the electric charge alone. Some years ago Dr. W. F. G. Swann expressed to me the desirability of calculating the radiation from an electron which rotates about its axis like a planet while describing a circular orbit. If such an electron can be treated as a magneton to a first approximation the above result is applicable. The fact that the radiation is reduced by rotation may indicate that revolving electrons do rotate.

4. A similar calculation for the case in which the axis of the magneton is tangential to the path indicates that the rate of radiation is greater than that from the electric charge alone.

5. It should be noticed that if $v' = v'' = g'' = 0$ the expression for I vanishes. Hence when an electromagnetic pole and an electromagnetic doublet move together with constant velocity along a rectilinear path, it is possible for the moment of the doublet to change at a constant rate without there being any radiation of energy.

[1] *Electromagnetic Radiation*, Cambr. Univ. Press, 1912: *London, Phil. Mag.* (Ser. 6), 36, 1918, 243.

ON THE DISTRIBUTION OF THE APHELIA OF THE SECONDARY BODIES OF THE SOLAR SYSTEM

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The finding of a dependence of orbital eccentricity upon the relative masses of the components led to an examination of the directions of the aphelia of the secondary bodies of the solar system, a resumé of which is the principal object of the present paper.

There is little to guide us as yet in the interpretation of such a dependence. The first explanation which suggests itself is that it is a residual effect of capture. In the comets we have a very exaggerated effect of such a dependence. Their eccentricities are essentially unity and whether they come from interstellar space or from the outer planetary regions they are, for the few journeys which they perform about the sun, captured bodies. This is with reference to the comets with very long period and those with sensibly parabolic orbits. The relations of the orbits of those with comparatively short periods to the outer (and larger) planets establishes the fact that such comets have been captured for the sun by these different planets. We have, therefore, in the solar system at least one class of bodies which has been captured. A study of the orbital characteristics of these in connection with the other secondary bodies (having the sun also for primary) should disclose any similarities which may be significant.

One fact which stands out prominently in connection with this dependence of orbital eccentricity upon relative mass appears to be significant. There are two possibilities aside from that of *no* preference

of eccentricity for relative mass: (a) the preference may be of larger eccentricities for systems with the *smaller* relative masses for the secondaries, or it may (b) be the reverse, a preference of the larger eccentricities for the systems with the *larger* relative masses for the secondaries.

The first of these possibilities is the one of which such widely distributed evidence has been found. Such a condition appears to result naturally from capture.

The second possibility (b), on the other hand, could not result from capture, nor does evidence of such a condition appear to prevail in nature.

It cannot but be regarded as significant, therefore, that of three possibilities the one which is actually found to prevail in nature is the one which would result directly from capture. The evidence is not sufficient to establish from the observed preference the reverse reasoning, viz., that because of this observed preference it must be concluded that binary and planetary systems have resulted from capture. It must be admitted, however, that this observational evidence, which appears to be stronger than any as yet adduced for a process of fission, places a theory of capture in a new light, and requires that it be seriously considered, not only for the solar system but for stellar systems as well.

If the secondary bodies of the solar system have been captured, and providing no force has seriously disturbed the major axes of their orbits since, or disturbed them all equally, we might expect to find a preference of the directions of their aphelia for some particular region of sky. It does not necessarily follow that it would be the region of the apex of solar motion. Differences of direction or velocity in the *relative* motions of the sun and the secondary bodies previous to capture could conceivably cause an apparent preference of their aphelia for almost any part of the sky.

The following four diagrams give in condensed form the results of an examination of the aphelia of the 8 major planets, 821 minor planets, 45 comets with periods of less than 80 years and 364 comets with longer periods and essentially parabolic orbits.

It is impossible to give in a limited space a full summary of the considerations resulting from this examination which has included not only the directions of aphelia but the orbital inclinations, longitudes of the nodes, galactic relations, etc. A full account of the investigation will be published in another place.

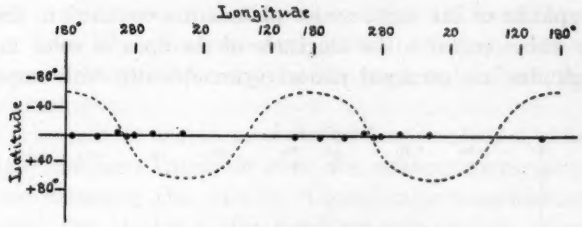

FIG. 1. APHELIA OF MAJOR PLANETS
Dotted line is the Galactic Plane

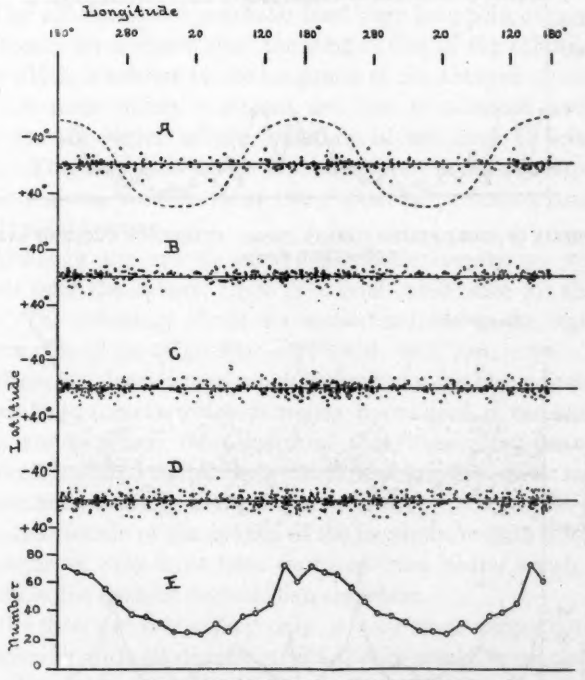

FIG. 2. APHELIA OF 821 MINOR PLANETS

A. Nos. 1 to 120 inclusive
B. Nos. 121 to 320 inclusive
C. Nos. 321 to 560 inclusive
D. Nos. 561 to 821 inclusive
E Frequency curve in longitude (all).

The following is a summary of the conclusions indicated by this investigation:

1. The axes of the orbits of the major planets, minor planets and short-period comets are all closely related to the ecliptic plane. This condition has been known in a general way.

2. The aphelia of the eight major planets are confined to the half of the sky whose center is the longitude of the apex of solar motion. Their longitudes are arranged almost symmetrically with respect to

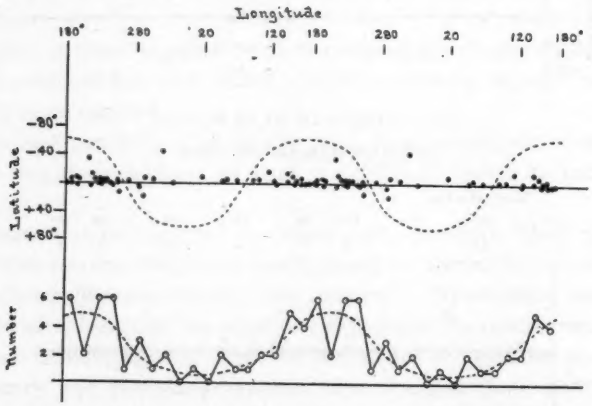

FIG. 3. APHELIA OF SHORT-PERIOD COMETS (UPPER). FREQUENCY CURVE OF SAME IN LONGITUDE (LOWER)

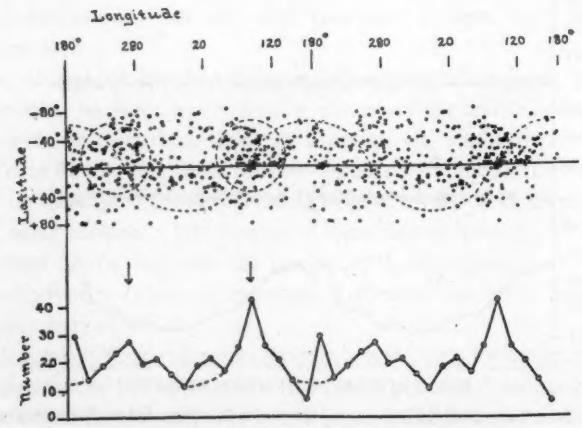

FIG. 4. APHELIA OF 364 PARABOLIC AND LONG-PERIOD COMETS. B. C. 371 TO A. D. 1914. FREQUENCY CURVE IN LONGITUDE (LOWER)

that point. The center of preference in direction of the aphelia of these bodies is, therefore, the intersection of the ecliptic and galactic planes.

3. The aphelia of the minor planets show a consistent and strong preference for longitudes in the region of 195°. The curve of preference

is closely a sine curve with the maximum at 195° and the minimum at 15°. There are indications of variations of a shorter period superposed on this curve which can be explained by more or less harmonic gaps at certain longitudes.

4. The aphelia of the 45 short-period comets show a preference for essentially the same longitude as do the minor planets, viz., 195°, the minimum occurring also at 15°. Considerable irregularities exist in this curve, due largely to the small amount of data. Some of the irregularities, however, appear to exist in the curves of parabolic comets also. The general form of the curve is none the less well marked.

5. The aphelia of the parabolic (and very long-period) comets show a preference for a region near the intersection of the ecliptic with the galaxy which is nearest to the longitude of the antapex of solar motion (90°). A more widely scattered and less pronounced preference is shown for the region of the longitude of the apex of solar motion (270°). There appears to be a tendency to avoid the intermediate galactic regions, between these two regions of preference and also the galactic polar regions.

6. Although the aphelia of the parabolic comets are widely distributed over the sphere, there is a small preference for the ecliptic plane. The deficiency of aphelia is most marked in the region of the northern pole of the ecliptic ($\alpha = 18^h$, $\delta = +67°$).

7. There is a deficiency of small inclinations (to the ecliptic) among the parabolic comets which is nearly normalized if the short-period comets are included. This indicates that these short-period comets have been captured out of the general herd and that their inclinations have not been radically changed since capture.

The distribution of the aphelia of the parabolic comets is what might be expected if they have been captured from bodies which are more plentiful in the galactic regions than elsewhere.

On the theory of probability only a very small proportion out of a mass moving with all directions and speeds would come close enough to be observable from the earth, except such as were moving with nearly the same speed as the Sun when they came within the distance of effective gravitational action. All of such, except those changed into small ellipses by planetary perturbation, will have sensibly parabolic motion. Preponderance of aphelia in the southern ecliptic hemisphere as well as the southerly ecliptic deviation of the maxima near the longitudes of the solar apex and antapex, indicate some sort of preferential motion of these bodies with respect to the Sun before

capture. The general preference of the aphelia of these comets for the longitudes of the solar apex and antapex can scarcely be a relation other than to the sun's motion among the stars. The greater condensation and preference for the longitude of the antapex appears to be real. Such a condition might perhaps follow if the cometary bodies were moving in general more rapidly than the Sun.

To summarize, the short-period comets show a preference of their aphelia for the same ecliptic longitudes as do the asteroids, but radically different from the distribution of the parabolic comets and those of very long period. As the short-period comets are conceded to be captures, has a similar preference of aphelia in the case of the asteroids also resulted from capture or is it due to some other cause? It is conceivable that such a preference might result from gravitational effects of the stellar system. If so why do the major planets show a different preference? The longitude of Jupiter's aphelion is at present approximately 192°, sensibly that of the preferences of the minor planets and short-period comets. Is there any connection? The moderate eccentricity of Jupiter's present orbit does not tend to strengthen a belief in such a relation.

It seems rather more than coincidence that the aphelia of all of the major planets are confined to one half of the sky and that the center of preference coincides so closely with the longitude of the sun's direction of motion in space, notwithstanding the observed revolutions of their lines of apsides, which differ greatly among themselves. Can it be that there are compensations or that our one or two centuries of (accurate) observations are insufficient to disclose gravitational or other effects of the stellar system? or does the mean of the planetary longitudes follow a changing direction of solar motion?

It is perhaps possible to believe that the observed preference of the major planets is coincidence, but not so the preferences of the minor planets and the comets. They are too numerous and their preferences too consistent. These unquestionably indicate a common underlying cause.

The parabolic comets show a very sharp maximum of aphelia at approximately 190° (fig. 4) preceding the one at 270° in some such way as the one at 20° precedes the strongest maximum of all at 90°. A division of the data into two groups shows the same peculiarities, indicating that it is real. This sharp maximum at 190° is perhaps significant, in connection with the similar preference of the minor planets and short-period comets for the same region.

This investigation has suggested a possible physical cause for the position of the ecliptic plane which, so far as I am aware, has not been satisfactorily accounted for. It is purely speculative and I know of no test which would be adequate, particularly as it rests upon the theory of capture which is far from being established. This explanation was suggested by the fact that the ecliptic intersects the galaxy not far from the extremities of the axis of preferential motion. There is some evidence that the two star streams may be in reality parts of a *curved* stream, the so-called axis of preferential motion being in reality a tangent to that curve. In my opinion such a conception best satisfies the observed conditions. Radial velocities indicate maxima to the *south* of both extremities of the so-called ellipsoidal axis. This brings the intersection near 18^h in the general region of what appears to be one star stream.

The centers of preference of the parabolic comets near the longitudes of the apex and antapex of solar motion are also both some 10° or 15° south of the ecliptic. If now the planets were captured, it is not difficult to outline a cause for the position of the ecliptic. Bodies moving with a stellar stream (not to be more definite at present) might be picked up by the Sun and if acted upon by a resisting medium, have their orbits rendered nearly circular. By mutual perturbation their orbits would tend to assume a common or 'invariable plane.' The addition of the masses of the satellites, comets and asteroids would not greatly disturb this plane, once established. This accounts for only one coordinate of the plane. The other we must suppose to be the mean of the group and as far as we know, at present, accidental.

This hypothesis is admittedly speculative and further discussion at this time does not seem warranted.

DISCOIDAL STRUCTURE OF THE LITHOSPHERE[1]

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Underlying the thesis here presented are the following postulates:

a. The lithosphere is heterogeneous as to density;

b. The lighter and heavier masses are of notable dimensions, varying from a hundred to several thousand kilometers in diameter;

c. Both lighter and heavier masses are distributed throughout continental areas and oceanic basins, but the lighter masses prevail in the continents and the heavier masses beneath the oceans;

d. The prevailing structure of deep-seated rocks is a foliated one and is the result of crystallization in response to appropriate conditions of stress and temperature;

e. Stresses competent to orient foliation in the elastic earth are set up by the effort of lighter and heavier masses to assume positions of isostatic equilibrium. These stresses are general, but they are localized and intensified by unloading and loading of adjacent areas through the process of erosion.

f. Periodic rises of temperature, local recrystallization, and local fusion of moderately deep-seated rocks result from the outward flow of the internal heat of the earth.

g. The strength of rocks increases with increasing pressure, as shown by Adams, but not as rapidly as the load increases from the surface of the earth downward. There is, therefore, a level at which the load equals the strength of any rock. Below that level for the strongest rock the earth is a potentially failing structure, which will fail by recrystallizing or by shearing under a differential stress which equals the critical load.

h. High temperature and fusion are potent factors in producing failure of the rigid lithosphere under its own weight.

It is obvious that these postulates rest upon the work of many investigators, among whom are Hayford and Bowie on gravitation, Dutton, Gilbert, and Barrell on the strength of the lithosphere, Van Hise, Leith, Johnson, and Bowen on schistosity, and Chamberlin on the origin of the earth and its internal heat. The contribution which this paper seeks to make is one toward an explanation of the mechanics of the lithosphere. Its conclusions, if valid, bear upon the growth of mountains, the deformation of superficial rocks, the distribution of volcanoes, the connections between continents, and the permanence of ocean basins.

The argument which leads to the recognition of the discoidal structure of the lithosphere proceeds as follows:

From an early stage of the growth of the earth onward the lighter and heavier masses of the elastic lithosphere have tended toward an equilibrium which may be described as an equilibrium in isostatic adjustment. It is conceivable that the departure from equilibrium may at some time have been such that the stresses set up in the lithosphere were adequate to cause flow and mass movement, until a close approach

to complete isostatic equilibrium was reached. But the present departure from equilibrium is not of such a magnitude, nor is it likely that there has been so great a departure during any stage of geologic time known to us. The stresses due to differences in weight of adjacent masses of lighter and denser material, even when they are intensified by erosion of broad surfaces to a peneplain and by correlative loading of basin margins, are regarded as quite incompetent to overcome the strength of underlying rock masses, whose rigidity increases with load.

Nevertheless, the effort toward isostatic equilibrium produces non-uniform stresses which are long enduring and persistent in direction. In any mass which was lighter than its environment the direction of least stress would be vertically upward and the directions of greater stresses would be horizontal. Under any mass which was heavier than its environment the direction of greatest stress would be vertical and the directions of least stresses would be horizontal. If the lighter and heavier masses be in elastic continuity with one another, the upward stress in the former and the horizontal stress under the latter would combine to form a resultant which would rise in a curve from below upward. Thus the zone embracing the adjacent parts of both columns would be subject to a curved stress, which would proceed outward and upward from under the heavier mass. Stresses of this character appear to be a necessary result of a lack of equilibrium among the masses or elements of the elastic lithosphere.

The law of crystalline orientation in response to stress, as developed by Van Hise, Leith, and Bowen, requires that foliation shall grow in the direction of least stress. If, then, rocks crystallize or recrystallize, through the influence of rising temperature and altering solutions, in adjacent masses which are pervaded by the curved stress due to isostatic inequilibrium, the resulting foliation should be oriented in a curve corresponding to that stress. The foliation should be horizontal under the heavier element, should approach verticality under the lighter element, and should curve upward continuously from under the former toward the surface of the latter.

The quantitative value of the least stress which will orient foliation under the exceedingly complex conditions of metamorphism has not been determined. It must not be confused with the differential stress required to produce mechanical flowage. Adams has demonstrated the latter to be very great. The former is a stress which is within the elastic limit and may safely be regarded as relatively very small.

More important than amount of stress are the characteristics of non-uniformity, as distinguished from hydrostatic pressure, persistence

in direction, and continuity of action during prolonged periods. These conditions are regarded as giving effectiveness to the stresses propagated in the elastic, rigid rocks far beneath the surface by imperfections of isostatic equilibrium.

The postulated directive stresses tending to orient foliation in curved surfaces have coexisted with conditions favorable to recrystallization since an early stage in the growth of the earth, under the planitesimal hypothesis. Under that hypothesis any resulting structure and its effects were characteristic of the growing earth and still pervade its mass from near the surface to the depth to which crystalline textures endure. Also under any other hypothesis of the earth's past history, which recognizes the effects of heterogeneity as to density and the periodicity of dynamo-metamorphism, structures involving curved foliation should result from isostatic stresses.

Upon the preceding reasoning is based the postulate of a discoidal structure of the lithosphere.

The condition for the development of curved surfaces of foliation exists wherever isostatic equilibrium is imperfect. It is obviously most effective where erosion and sedimentation are active. It is not limited to any particular section of the border zone of any lighter or heavier mass which is not in equilibrium, but extends around each such heavier mass and invades the area of each such lighter mass. Since the curves of strain and the resultant foliation dip under the heavier masses from all sides, those masses have a discoidal structure. They may be designated as discs. They are regarded as composed of a series of foliae, which are convex downward and which make up the mass of the disc to a depth of several hundred kilometers. Their outcropping margins will include wider or narrower zones of the lighter structural masses. Between the discs may lie masses which, by reason of the fact that they do not pertain to the heavier disc-shaped masses, belong to the lighter masses. These may be called interdiscs. Their cross section in a vertical plane would present a more or less broadly truncated triangle, whose truncated upper surface would vary from sub-continental to merely isthmian widths.

Thus the lithosphere would superficially exhibit three structural types: (1) the surfaces of the heavier discs, depressed by gravitative adjustment and more or less deeply covered with water or continental sediments. The dominant characteristic of the underlying masses is high density. It is a permanent characteristic and its effects are permanent and continuous. They should not be confused with the temporary effects of orogenic movements, which may also result in super-

ficial depressions; (2) the surfaces of the lighter interdiscs, riding high and covered with the sediments of epicontinental seas, or exhibiting an approximately vertical foliation where denuded to metamorphic rocks; (3) the marginal zones of discs and interdiscs, characterized by extrusion of igneous rocks and orogenic disturbances. Where a broad light mass or structural element lies adjacent to a broad heavy element, as along the Pacific Coast of North America, the zone of marginal deformation may be very wide. Where, on the contrary, two heavy elements are nearly tangent, the zone of disturbance may be exceedingly narrow, as the Isthmus of Panama.

The hypothesis of discoidal structure may best be tested by the facts of the earth's major features and their relations to igneous intrusion and extrusion, to regional, periodic diastrophism, and to orogeny. Certain lines of investigation are suggested in what follows.

Are igneous rocks distributed around deeps, as would be required by the discoidal hypothesis?—Let it be assumed that magmas originate, as postulated by Chamberlin, through local fusion in the outer few hundred kilometers of the lithosphere. Molten material gathering in the foliae beneath a disc would be guided by the foliation toward the surface and would appear in or near the outcrop of the margin of the disc. By postulate any disc is indicated on the surface by an oceanic deep or a continental basin. The relation of volcanoes and also of geologically recent batholiths to oceanic margins has long been recognized. As bathymetric observations accumulate a more detailed relation to individual deeps is noticeable. The most conspicuous examples are the Windward Islands, sometimes called the "Necklace of the Caribbean," and the Alaskan, Aleutian, and Japanese volcanic chain.

Does diastrophism exhibit that periodicity with reference to the larger deeps which we should expect if each deep constitutes a structural unit?—It is well established that the opposite coasts of the North Atlantic have periodically suffered similar and contemporaneous deformation, and the same is true of the encircling lands around the Pacific Ocean.

Have the diastrophic activities of separate great deeps been non-contemporaneous to the extent which would give to each an independent character peculiar to itself?—That the diastrophic activities of the Atlantic and Pacific basins are incommensurate on the geologic time scale is a fact which constitutes the gravest difficulty in the assumption of universal periodicity. If diastrophism be due to an ultimate cause, operating periodically upon the inner mass of the earth, such as the accumulation of internal heat, an explanation of the diverse periodicity of different oceanic basins must be sought through the mechanism

which registers the effects at the surface more quickly in one region or more slowly in another, as the case may be.

Do the mechanics of deformation of the lithosphere, as expressed in the effects of compression, conform to the structures which should follow from the postulated discoidal structure?—Under general tangential stress a spherical shell composed of discs and interdiscs should shorten by riding up of the margins of the discs upon the interdiscs. If the disc were competent to sustain itself by partial bridging, its whole mass would rise; if, as must generally if not always have been the case in the lithosphere, the disc be too wide and too weak to support itself, the margins only would be raised, the mass bending or fracturing as conditions demanded. The marginal zones should exhibit over-thrusts from the basin side and incidental phenomena of folding with minor over- and under thrusting. The facts of mountain structure appear to conform to these requirements. The folded and overthrust structures of the Appalachians and the faulted and upthrust structures of the Pacific ranges of California appear to find an intelligible explanation on the basis of the discoidal theory.

Do the trend lines of mountain chains conform in plan to the postulated discoidal structure?—To answer this question in detail will require an analysis of the earth's surface into discs and interdiscs, but the trend lines of the mountain chains of Europe, Asia, and the two Americas, as developed by Suess, do conform in plan to the obvious depressions within and around those continents. It must be recognized, however, that a long narrow depression filled with sediment may be so compressed and displaced as to become part of the continental interdisc. This is a well known geological occurrence, the transformation of a geosyncline into a mountain range, but it cannot in every such case be assumed that the geosyncline corresponded to a dense underbody. Other mechanical conditions may produce subsidence. With this reservation, however, I believe that the directrices of mountain chains can be shown to wind around discoidal masses.

Does the discoidal theory suggest any solution for the temporary intercontinental land connections which are demonstrated by biologic evidence?—The present isthmian connection between North and South America may be regarded as a typical case of intercontinental bond established by marginal uplift and extrusion of igneous rocks between the Caribbean and eastern Pacific discs. Similar connections may readily be traced between South America and Africa by two different routes. A connection between North America and Europe, via Greenland, Iceland, and England, such as has often been suggested, finds a rational

explanation through its relation to the deeps of the North Atlantic. Oceanica may be intelligently linked up and explained.

If the establishment of isthmian connections be explicable according to the discoidal theory, can their subsequent submergence also be accounted for?—According to the discoidal theory, the source of very large masses of igneous rocks lies in the base of the discs. The extrusion of the melt is accompanied by subsidence of the central portion of the disc, which in turn and in time may be succeeded by slipping down of the elevated margins. The mechanism appears to be workable. It requires, however, fuller elaboration than can here be given to elucidate it. These concepts are obviously consistent with the permanency of oceanic basins and continental masses.

In addition to the tests proposed by these general questions, the theory should be tried by the more exacting requirements of the mechanics of deformation. This has been attempted and not without reasonably satisfactory results. The structure of the Pacific ranges of California has been discussed before the Geological Society of America and the LeConte Club of California, and the paper will shortly be offered for publication.

[1] Abstract of a paper presented to the Geological Society of America at the Pasadena Meeting of the Cordilleran Section.

RECENT SIMULTANEOUS MEASUREMENTS OF THE SOLAR CONSTANT OF RADIATION AT MOUNT WILSON, CALIFORNIA, AND CALAMA, CHILE

By C. G. Abbot

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Read before the Academy, April 29, 1919

For the past fourteen years, with the exception of the year 1907, the Smithsonian Astrophysical Observatory has observed the solar radiation at its station on Mount Wilson, California, altitude 5700 feet. Measurements are made by means of the pyrheliometer and spectro-bolometer at different hours of the day, so chosen that the intensity of the solar radiation as it would be outside the atmosphere at mean solar distance may be computed therefrom. During the earlier years of the investigation it appeared that the values found vary from day to day, as well as from year to year. In other words, it seemed to be indicated that the sun is a variable star, having a two-fold variation. First, a varia-

tion of long period, having a range of from 3% to 5% during the period of the sunspot cycle. Second, a short interval irregular variation running its course in intervals of a few days, weeks or months, and having a range often as great as 3%, sometimes as great as 7% and even in exceptional cases of 10%.

In the years 1911 and 1912, the reality of this supposed short irregular variability of the sun was tested by observing simultaneously at Mount Wilson and at Bassour, in Algeria, a station about 50 miles south of Algiers, situated at an elevation of about 3600 feet. The results of the two stations on the whole supported one another and indicated that the variations were due to causes outside the earth's atmosphere.

In the year 1918, the Smithsonian Institution sent an expedition, at the cost of the Hodgkins Fund, to Calama, Chile, to obtain during a term of years, daily measurements as far as possible of the intensity of the solar radiation. The station is situated about 150 miles northeast of Antofagasta at an elevation of 7500 feet, in one of the most cloudless regions of the world, where rainfall is almost unknown. The expedition is in charge of Mr. A. F. Moore, Director, assisted by Mr. L. H. Abbot. The observers began actual observing on July 27, and in the six months next following had observed on about 70% of the days.

During a part of this interval measurements were made on Mount Wilson by my colleague, Mr. Aldrich, and the computations have now reached such a stage of advancement that we are beginning to be able to tell that the two stations mutually support one another as to the short irregular periodicity of the sun.

In order to make a proper comparison it is evidently not necessary that the days observed should be consecutive but only that satisfactory observations should be obtained on each day of the comparison at both stations. In order to exhibit the dependence in the clearest possible manner, we may indicate Calama values by abscissae or horizontal distances, and Mount Wilson values by ordinates or vertical distances. If the measurements were without error and there was no variation of the sun, evidently all results should fall upon the same point, whether observed at one station or the other. But if there was a true variation of the sun, the point would be stretched out into a line at 45 degrees to the axes. As all measurements are subject to error it is not to be hoped that the several points corresponding to the several days of observation will all be found strictly upon such a line, but they ought to arrange themselves about it in such a way that the line will give the best mean representation.

This condition is well fulfilled. A change of 1% or less would suffice to bring almost every point on the line. Accordingly we may conclude that the spectro-bolometric determinations of the solar constant of radiation made at Mount Wilson and Calama mutually support one another and indicate a variability of the solar radiation. Since the two stations are situated in different hemispheres of the earth, separated by a distance of upwards of 5000 miles, it must be conceded that the result strongly confirms our view of the short period variability of the sun.

Investigations have already been made by Dr. Clayton and others on the correlation of terrestrial temperatures and pressures with these supposed changes in the sun, and affirmative results have been obtained by them to the question: Do the variations in the sun noticeably affect the terrestrial weather conditions? Dr. Clayton is so strongly convinced of the dependence between the weather and the solar radiation that at his desire the Argentine Government is obtaining daily telegraphic solar radiation reports from the station at Calama on which are based forecasts of the temperature in Argentina. Hitherto the correspondence has been very good, so that Dr. Clayton is very sanguine that a great improvement in forecasting will result from these solar radiation observations. The Brazilian Government has also, at his recommendation, begun comparisons.

If the further studies of the correlations between terrestrial temperatures and solar variations yield similar affirmative results, it will probably be necessary within a year or two to undertake the equipment of several additional solar radiation observing stations in the most cloudless regions of the earth. There should be not less than three additional solar radiation stations, and the cost of conducting them and the station at Calama would absorb an income of from thirty to forty thousand dollars per annum. It is to be hoped that funds for this purpose may become available to the Smithsonian Institution, for with the experience gained by the staff of the Institution it would be able to carry on the research in a way to obtain homogeneous and accurate results.

The choice of sites for additional solar radiation stations involves investigation as to the most cloudless regions of the earth. None of the stations hitherto occupied, not even that at Calama, is as satisfactory as we could wish for. The most serious enemy to the research is the presence of cirrus clouds, or any other form of cloudiness, and it is a grave problem where in the world these are least numerous. The comparison of the observations at the two stations in 1918 shows that no inconsiderable number of the days in which observations were made were unsuitable for the purpose on account of the changes of the atmos-

pheric transparency during the two or three hours necessary to the determination. It is this defect of the uniformity of the atmospheric transparency which requires us to make the measurements at so many stations in order to get a satisfactory result.

Both the Mount Wilson and Calama observations indicate that the solar radiation was above its normal in the year 1918, having an average value of about 1.95 calories per square centimeter per minute. The mean value for many years was 1.93.

ROTATING PROJECTILES FROM SMOOTH-BORE GUNS

By C. G. Abbot

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In the late war much use was made of trench mortars. This kind of ordnance consists essentially of a smooth tube with a firing pin at the bottom. The projectile carries a shotgun shell at the rear. It is dropped down the smooth-bore barrel by the soldier and the firing pin explodes the primer on the shotgun shell. This in its turn explodes the charge of propellant which throws the shell into the enemy's lines. Many of the trench mortar shells tumbled end over end as they went over, but some were provided with fins which retained them in approximately steady flight. However the trajectory of these shells was by no means ideal.

It occurred to me to try to secure sufficient rotation to produce steady flight by means of the turbine principle applied to the shells. Unfortunately the research was proposed to me only a few days before the armistice was declared so that by the time preliminary tests were made it was too late for the device to be of war service. However, the results appeared to be so promising that I have made numerous further experiments with a smooth-bore musket of 0.9 inch diameter which had seen service in the Civil War.

The elongated ojival projectile was made in two parts in my experiments, the one a tough steel rear part chambered out to contain the propellant, the other an aluminum nose provided with a steel plug at the rear for screwing into the steel part of the projectile. The steel plug serves the double purpose of closing the chamber and attaching it to the front part. With the aluminium part in front such projectiles are particularly well calculated for tumbling, and invariably did so when fired in the usual manner. At the extreme rear of the steel part of the

projectile a plurality of bell-shaped bores were provided, leading tangentially from the chamber within the projectile to the rear where they emerged into the barrel of the gun. On screwing the aluminum nose with its steel boss into the opening of the chamber the whole presented the appearance of an ordinary elongated projectile except for the bell-mouthed bores visible at the rear. When the gun is fired the propellant within the chamber is ignited by priming contained in the tangential apertures.

The outflow of the gases through the bell-mouthed bores, while they would give some rotation, owing to their friction within the chamber, would fail to give sufficient rotation if it were not that steel pins are inserted across the chamber to form a sort of baffle, so that the outflowing gases may react against this baffle and so tend more strongly to rotate the projectile.

It was feared that the combustion would not be completed, or at least that the gases would still remain in the chamber at high condensation at the time when the projectile left the muzzle of the gun. If this were the case it might easily be that owing to some slightly unsymmetrical construction of the bell-mouthed bores some tendency to deflect the flight of the projectile would be caused thereby by the delayed outrush of the residue of gases. In order to avoid this, the end of the barrel was continued by a tube extending some 10 inches beyond the former muzzle and at the rear of the tube where it joined the barrel there were drilled a number of holes so that the gases might escape therefrom, and relieve the pressure within the chamber of the projectile while still it was subject to guidance from the lengthened barrel of the gun.

Very satisfactory results have been obtained in firing these special projectiles. This method of securing rotation of projectiles appears to be suitable not only for trench warfare but for all varieties of ordnance larger than 1 inch in diameter. Rotations as great as one rotation in 18 calibers have been obtained thereby. It is customary to employ about one rotation in 30 calibers in the ordnance of Europe. The United States employs one rotation in 25 calibers for a large number of its heavy guns. My experiments seem to show that one rotation in 40 or 50 calibers is sufficient to prevent any appreciable deflection of the shells in flight.

The experiments have been conducted within the enclosure of the Astrophysical Observatory, so that it was impracticable to employ heavy charges so as to get very high speeds of flight, but within the range of the experiments I have found by shooting through a succession of paper screens and observing the holes with a theodolite that within the length

of the observatory enclosure there is no measurable lateral deflection of the projectile while in flight. Truly round holes are always left by the rotating projectiles. Very striking results were obtained in an experiment made in Virginia in the presence of a number of Ordnance Officers. Two successive shots were fired at a target in one of which the projectile was made to rotate by means of the included charge and in the other of which the projectile was fired by powder outside of itself. In the one an excellent hit was made, leaving a true round hole, while in the other it happened that in the tumbling of the shell in the air it reached the target exactly broadside. The officers were naturally much impressed by this striking exhibit.

As in most of the investigations we undertake, the same general idea had long before occurred to others. Patents for somewhat similar devices were granted as long ago as the Civil War. So far as I am aware, however, no one hitherto has attained so good a measure of success in applying the turbine principle to projectiles.

MEANS OF MEASURING THE SPEED OF PROJECTILES IN FLIGHT

By C. G. Abbot

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In the course of the experiments on rotating projectiles from smooth bore guns I desired to measure the speed of flight in some instances, and as I lacked the usual chronographic apparatus employed by artillerists the following substitute occurred to me and proved very satisfactory after a few trials.

The projectile was fired obliquely across a horizontal beam of light reflected into the observatory from the siderostat. The solar beam was entirely cut off by a diaphragm at the wall of the observatory, except as it passed through two round holes about 6 inches apart. The shot was fired through the left hand one of these holes. About 2 meters inside of the observatory another smaller diaphragm with a small aperture in it was placed opposite to the second or right hand hole. The direction of the gun was so arranged that the shot passed through the hole in this second diaphragm also. About a meter beyond the second diaphragm was placed a double tinfoil screen, the two tinfoils being separated by a sheet of cardboard and connected by wires respectively to a circuit con-

taining an electromagnet which operated a shutter for the purpose which I am about to describe. Opaque cards were placed to cover the left hand hole in the first diaphragm through which the shot was fired and to cover the hole in the second diaphragm through which the shot passed after having accomplished two meters more of its flight. The passage of the shot through the tinfoils beyond closed the electric circuit and operated the shutter which has been mentioned.

The beam from the left hand hole of the first diaphragm passed to a mirror at 45° which reflected it at right angles upon a photographic plate caused to rotate in its own plane by means of an electric motor. The beam from the right hand hole of the first diaphragm, after passing through the second diaphragm was also reflected by the same mirror upon the same photographic plate but diametrically opposite to the first beam with respect to the center about which the plate rotated in its own plane. Between the mirror and the photographic plate was a long slit which restricted the two beams of light to mere streaks extending respectively from the center to the right hand and to the left hand of the photographic plate. Just above the long slit was placed the shutter controlled by the electromagnet.

It will readily be seen that when the gun was fired, first a beam from the left hand hole in the first diaphragm passed to the mirror and was reflected upon the rotating plate. After the projectile had covered the two meters leading to the second diaphragm, the other hole was opened and the second beam began to print upon the other half of the photographic plate. Immediately thereafter the electric circuit was closed, the shutter consequently closed, and so the photographic action stopped before the plate had made a half revolution. The delay in the electric action had no influence on the work provided the shutter closed before the plate made a half rotation. After development the blackened parts of the photographic plate were found to indicate a certain angle through which it had rotated between the time when the projectile opened the first aperture and the time when it opened the second aperture. Having determined the speed of the photographic plate by measuring the speed of the electric motor, the velocity of the projectile became known.

It is easy by this means to determine with some accuracy very great velocities of translation within a very short path. Even one meter path would be quite sufficient.